Acta Polytechnica **56**(2):81–87, 2016
doi:10.14311/AP.2016.56.0081

available online at http://ojs.cvut.cz/ojs/index.php/ap

APPLICATION OF DIMENSIONAL ANALYSIS IN DETERMINING THE HEAT LOSS IN DISTRICT HEATING SYSTEMS

Mária Čarnogurská[a,*], Romana Dobáková[a], Miroslav Příhoda[b]

[a] *Technical University of Košice, Faculty of Mechanical Engineering, Department of Power Engineering, Vysokoškolská 4, 042 00 Košice, Slovakia*

[b] *VŠB – Technical University of Ostrava, Faculty of Metallurgy and Materials Engineering, Department of Thermal Engineering, 17. listopadu 15, 708 33 Ostrava-Poruba, Czech Republic*

[*] corresponding author: maria.carnogurska@tuke.sk

Abstract. Existing procedures for determining the heat loss in heat distribution use balance relations, or rather they are based on the theory of heat and mass transfer. Dimensional analysis enables a new point of view that demonstrated the functionality of heat loss from selected physical quantities that contribute to heat loss. The solution provides one complex criterion and four simplex criteria of similarity. Transforming these criteria of similarity leads to the formulation of the functional dependence of only two criteria, on the basis of which the amount of heat loss can be determined. The resulting criterial dependency is simple, and is calculated in this paper for a pipe one meter in length. The mathematic model for demonstrating the heat loss is of universal validity, and applies to a wide range of piping used for hot water distribution. However, for each nominal diameter of piping it is necessary to take into account the mutual dependency of the dimensionless arguments π_5 on π_1, the form of which is always different. In the paper, this dependency is demonstrated for two nominal diameters DN65 and DN125.

Keywords: heat loss; dimensional analysis; modelling.

1. Introduction

Diverse phenomena can be modelled by more or less known procedures and methods. In the area of heat flow and transmission, balance equations are used for an analytic solution [1–3]. For a numerical calculation, methods of finite elements, finite volumes and finite differences [4–6] are mostly used. An analytic method based on the dimensions of physical quantities, so-called dimensional analysis, offers a special type of solution. Over a long period of time, Čarnogurská and her co-workers have studied [7–12] the application of this methodology, which is a tool suitable for describing any phenomenon that currently does not have a mathematical formulation in the form of a differential equation or an empirical formula.

The existing procedures for determining heat loss (specific and total) on the basis of the balance method are related to measurements of thermal difference and the flow rate of a heat transmission medium in a distribution network. The detailed methodology can be found in [13–15]. The balance method requires excellent measuring equipment that enables water temperatures to be measured in hundredths of one degree, and such technology cannot be fixed in complicated central heating systems.

The disadvantages of the balance method for determining heat loss have pointed to the need to develop new methods for expressing the specific heat loss and the total heat loss in a simpler way that is applicable for network operators.

Dimensional analysis on the principle of the equation for dimensional homogeneity forms the basis for a newly-developed methodology for determining heat loss. Dimensional analysis is based on the Buckingham π-theorem. This theorem expresses that the number of dimensionless criteria i is equal to the number of all relevant dimensional quantities n reduced by the number of basic quantities r. In cases where some rows of the dimensional matrix are linearly dependent on each other, van Driest expressed the general formulation of the π-theorem: the number of dimensionless criteria is equal to the number of all the relevant dimensional quantities reduced by the rank of the matrix h, i.e., by the number of linear independent equations of the system [12, 16]. The outcome of calculating the heat loss by this method is a relation that is valid universally for all heat transmission networks, above the ground or underground, direct or reverse.

2. Study area

In the balance method, the heat loss (or the thermal power loss) is expressed in dependency on the water flow rate, the decrease in the water temperature in the monitored area of the piping, the speed of the flow, the discharge area and the water density.

In the application of dimensional analysis, the internal and external diameter of the piping, the external diameter of the insulation, the thermal conductivity of the insulation, the length of the piping, the speed of the water flow in the piping, the water temperature in the inlet point of the piping, and the ambient temperature are taken into consideration.

Quantity	Symbol	Dimension
Heat transmission medium temperature (water)	T_i	K
Temperature of the surroundings (air, soil)	T_e	K
Pipe internal diameter	d_1	m
Pipe external diameter	d_2	m
Insulation external diameter	d_3	m
Insulation thermal conductivity	λ_{ins}	$\text{W m}^{-1}\,\text{K}^{-1}$
Speed of water flow in the pipe	v	m s^{-1}
Pipe length	l	m
Heat loss	P	W

TABLE 1. Physical quantities used in constructing the model

3. Expressing heat loss by the balance method

Using the balance method, the thermal power loss of an insulated pipe can be determined by the following relation:

$$P = Q_m \cdot c \cdot \Delta t, \tag{1}$$

where Q_m is the water mass flow (kg s^{-1}), c is the water specific heat capacity ($\text{J kg}^{-1}\,\text{K}^{-1}$), and Δt is the decrease in the water temperature in the respective area (°C). The mass flow Q_m is set by

$$Q_m = \rho \cdot Q_V. \tag{2}$$

The following applies for the flow rate:

$$Q_V = v \cdot S, \tag{3}$$

where Q_V is the water flow rate (m s^{-1}), ρ is the water density (kg m^{-3}), v is the water flow speed in the piping (m s^{-1}), and S is the flow area of the pipe (m^2).

In addition to relation (1), the following relation can be used for calculating the power loss

$$P = q \cdot S_{\text{ins}} = q \cdot \pi \cdot d_3 \cdot l, \tag{4}$$

where q is the heat flow density on the surface of the pipe insulation (W m^{-2}), S_{ins} is the surface area of the pipe with the insulation (m^2), l is the length of the monitored part of the pipe (m), and d_3 is the external diameter of the insulation (m).

From the equality of relations (1) and (4), we can determine the specific heat loss (the heat flow density) for direct piping and also for reverse piping, whether above the ground or underground, depending on the increase in the water temperature between the inlet and outlet of the piping, in the form

$$q = \frac{Q_m \cdot c \cdot \Delta t}{\pi \cdot d_3 \cdot l}. \tag{5}$$

The relation between the heat flow density q and the linear heat flow density q_l is expressed by:

$$q = \frac{q_l}{\pi \cdot d_3}, \tag{6}$$

from where the specific heat loss

$$q_l = q \cdot \pi \cdot d_3. \tag{7}$$

By inserting relation (5) into (7) we get the formula for calculating the specific heat loss, as follows:

$$q_l = \frac{Q_m \cdot c \cdot \Delta t}{l} = Q_V \cdot \rho \cdot c \cdot \Delta t_{1\text{m}}, \tag{8}$$

where $\Delta t_{1\text{m}}$ is the temperature decrease in a pipe one meter in length.

4. Expressing heat loss by dimensional analysis

When using dimensional analysis for determining heat loss, it is necessary to select those physical quantities which definitely influence this phenomenon. If a quantity with no significant influence has been selected, it usually drops out from the solution spontaneously.

The quantities listed in Table 1 were selected for elaborating the mathematical model.

An analysis of these quantities shows that – out of five dimensions of physical quantities of equal value – the heat loss is described by a set of four simplexes (criteria of similarity) in the following form (these criteria of similarity are dimensionless):

$$\pi_1 = \frac{T_i}{T_e}, \quad \pi_2 = \frac{d_1}{l}, \quad \pi_3 = \frac{d_2}{l}, \quad \pi_4 = \frac{d_3}{l}. \tag{9}$$

Taking into consideration that the number of quantities used for elaborating the model in Table 1 is $n = 9$, and the number of dimensions determined by these SI system quantities is in total $r = 4$ (K, m, kg, s); the number of dimensionless arguments that describe the heat loss is defined by the subtraction $n - r = i$, i.e., 5 arguments. Another complex needs to be added to the four simplexes mentioned above. This complex results from the matrix form of the scheme of relevant quantities and their dimensions.

In line with the rules for the application of dimensional analysis, it is possible to create a complete physical equation from relevant quantities with different dimensions; the equation is in the following form:

$$\varphi(P, T_i, T_e, d_1, d_2, d_3, \lambda_{\mathrm{ins}}, \upsilon, l) = 0. \tag{10}$$

Equation (10) demonstrates the fifth dimensionless argument, for which the following scheme is applicable:

$$\pi_i = P^{x_1} \cdot T_i^{x_2} \cdot T_e^{x_3} \cdot d_1^{x_4} \cdot d_2^{x_5} \cdot d_3^{x_6} \cdot \lambda_{\mathrm{ins}}^{x_7} \cdot \upsilon^{x_8} \cdot l^{x_9}. \tag{11}$$

Dimensional matrix A has 9 columns for the basic units. The number of lines in the matrix corresponds to the number of basic units, i.e., 4. Its form is

$$\begin{array}{c} \\ \mathrm{kg} \\ \mathrm{m} \\ \mathrm{s} \\ \mathrm{K} \end{array} \begin{array}{c} \begin{array}{ccccccccc} P & T_i & T_e & d_1 & d_2 & d_3 & \lambda_{\mathrm{ins}} & \upsilon & l \end{array} \\ \left\Vert \begin{array}{ccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ -3 & 0 & 0 & 0 & 0 & 0 & -3 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \end{array} \right\Vert \end{array}. \tag{12}$$

The rectangular matrix (12) is split into two parts during the solution. The first part of the matrix ($\mathbf{P}$) has 8 columns, and 4 lines, where the lines must be selected in such a way that the determinant has a non-zero value ($\Delta_{\mathbf{P}} \neq 0$). Also, in line with this is the splitting of the unknown quantities vector x_i from Eq. (12). Its symbol is $\mathbf{R}$. For matrix $\mathbf{P}$ and the unknown quantities, the vector $\mathbf{R}$ relation (13) applies [9]:

$$\mathbf{P} \cdot \mathbf{R} = (-1) \cdot \mathbf{Q} \cdot \mathbf{S}, \tag{13}$$

where $\mathbf{Q}$ is a matrix vector with number of columns 1 and number of lines 4. $\mathbf{S}$ is the unknown quantity vector with number of columns 1 and number of lines 1. Equation (13), expressed in the form (12), can be detailed in the form of relation (14):

$$\left\Vert \begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -3 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \end{array} \right\Vert \cdot \left\Vert \begin{array}{c} x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{array} \right\Vert = (-1) \cdot \left\Vert \begin{array}{c} 1 \\ 2 \\ -3 \\ 0 \end{array} \right\Vert \cdot \Vert\, x_1 \,\Vert. \tag{14}$$

The solution of the system of linear equations that can be created from Eq. (14) enables us to calculate unknown x_1 to x_9. The wanted independent vector π for individual unknowns will be in the form

$$\pi = \begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{array} \\ \left\Vert \begin{array}{ccccccccc} 1 & -1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \end{array} \right\Vert \end{array}. \tag{15}$$

This independent vector represents the fifth dimensionless argument. In its numerator, there is the physical quantity, the exponent x of which is needed in Eq. (11). It has a positive value in expression (15). In the denominator, there are quantities that have a negative value in the exponent. The numeral character 1 in expression (15) means that the respective physical quantity is raised to the first power. The desired dimensionless argument therefore has the form

$$\pi_5 = \frac{P}{T_i \cdot \lambda_{\mathrm{ins}} \cdot l}. \tag{16}$$

From the solution, it is clear that the speed of the water flow is not present in the dimensionless argument π_5. Thus it is redundant in the selected relevant quantities. The heat loss of the piping does not depend on the speed of the water flow.

From the criteria of similarity that are obtained (relations (9)) and the criterion described by Eq. (16), the functional dependency in the dimensionless form will be formulated describing the heat loss, as follows:

$$\Psi(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0, \tag{17}$$

from where

$$\pi_5 = \Phi(\pi_1, \pi_2, \pi_3, \pi_4). \tag{18}$$

The form of the dependency of the dimensionless arguments can generally be expressed by:

$$\pi_5 = C \cdot \pi_1^n \cdot \pi_2^m \cdot \pi_3^o \cdot \pi_4^r. \tag{19}$$

For each type of pipe length l, the arguments π_2, π_3 and π_4 have constant values (hereinafter referred to as C). Under the condition $\pi_2 = C_1$, $\pi_3 = C_2$ and $\pi_4 = C_3$, Eq. (19) is modified into

$$\pi_5 = C \cdot \pi_1^n \cdot C_1^m \cdot C_2^o \cdot C_3^r, \tag{20}$$

where n, m, o, r are exponents, and C is a constant.

After a merge of all constants into one, referred to as K, relation (20) is simplified into

$$\pi_5 = K \cdot \pi_1^n, \tag{21}$$

where $K = C \cdot C_1 \cdot C_2 \cdot C_3$.

Equation (21) represents a power function. After it has been transformed into logarithmic coordinates, it indicates

$$\log \pi_5 = \log K + n \cdot \log \pi_1. \tag{22}$$

Locating constant K and regression coefficient n can only be determined from an experiment carried out in a specific heating network. Only data from long-term monitoring of the network (in all-year-round operation) are applicable, as only these data provide information on the impact of the ambient temperature on the total heat loss (summer and winter network operation).

From Equations (21) and (9), the relation for the power loss of the whole heating network can be derived in the following form:

$$P = K \cdot T_i^{1+n} \cdot \lambda_{\mathrm{ins}} \cdot l \cdot T_e^{-n}. \tag{23}$$

Thus, for the thermal power loss of piping one meter in length, the following will apply:

$$q_l = K \cdot T_i^{1+n} \cdot \lambda_{\mathrm{ins}} \cdot T_e^{-n}. \tag{24}$$

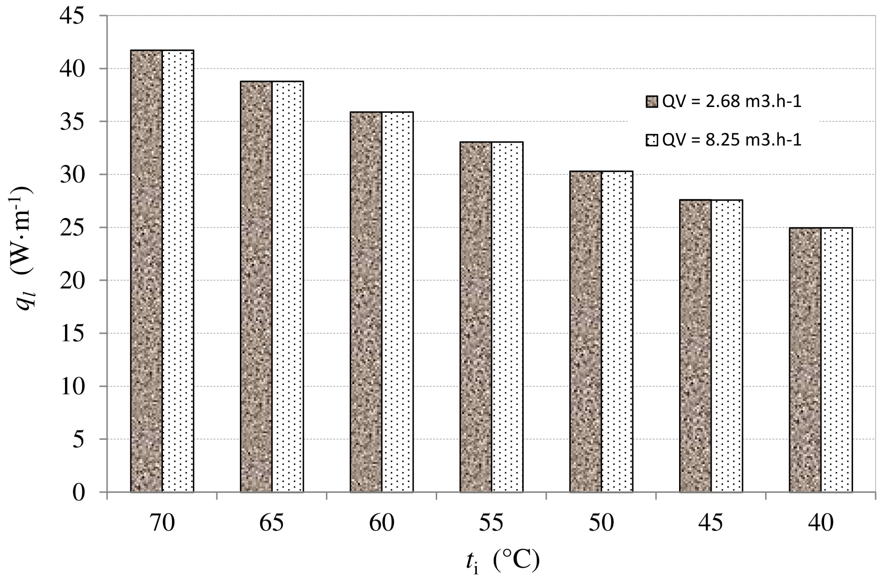

FIGURE 1. Course of the specific heat loss – balance method.

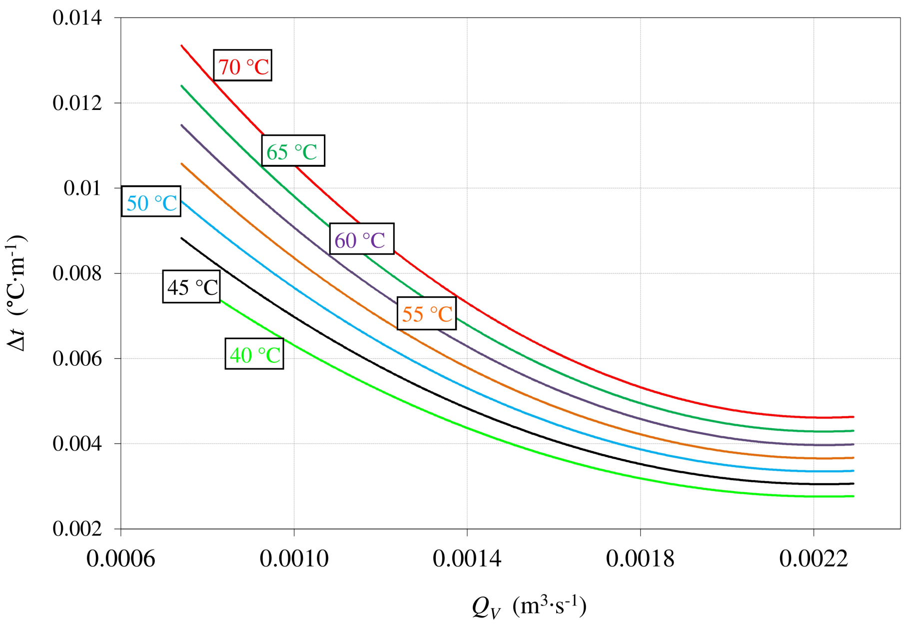

FIGURE 2. Course of Δt in dependence on the change in the flow rate for different temperatures of the transferred water.

5. EXPERIMENTAL RESEARCH ON HEAT LOSS

The experimental research was focused on the heating network above ground. Its nominal diameter is DN125, with external piping diameter $d_2 = 133$ mm, piping wall thickness $s = 3.6$ mm, insulation thickness $s_{\mathrm{ins}} = 33.5$ mm, length of the monitored network $l = 27$ m, median of the insulation thermal conductivity factor (as a function of the water temperature in the direct piping) $\lambda_{\mathrm{ins}} = 0.041\,\mathrm{W\,m^{-1}\,K^{-1}}$, the median of the insulation thermal conductivity factor (as a function of the water temperature in the reverse piping) $\lambda_{\mathrm{ins}} = 0.04\,\mathrm{W\,m^{-1}\,K^{-1}}$, the median of the heat transfer factor from the piping surface into the outer environment $\alpha_e = 3\,\mathrm{W\,m^{-2}\,K^{-1}}$. The temperature of the water coming into the direct piping was between 55 °C and 70 °C. The temperature of the water flowing out of the reverse piping was between 40 °C and 60 °C. The outside temperature was considered to be $t_e = -15$ °C. According to Standard STN EN 12831, this value represents the calculated he outside temperature in the city of Košice.

5.1. CALCULATING THE HEAT LOSS BY THE BALANCE METHOD

During the measurements, the flow rate was in two limit states:

- $Q_{V1} = 8.25\,\mathrm{m^3\,h^{-1}}$ – during the heating period;
- $Q_{V2} = 2.68\,\mathrm{m^3\,h^{-1}}$ – outside the heating period.

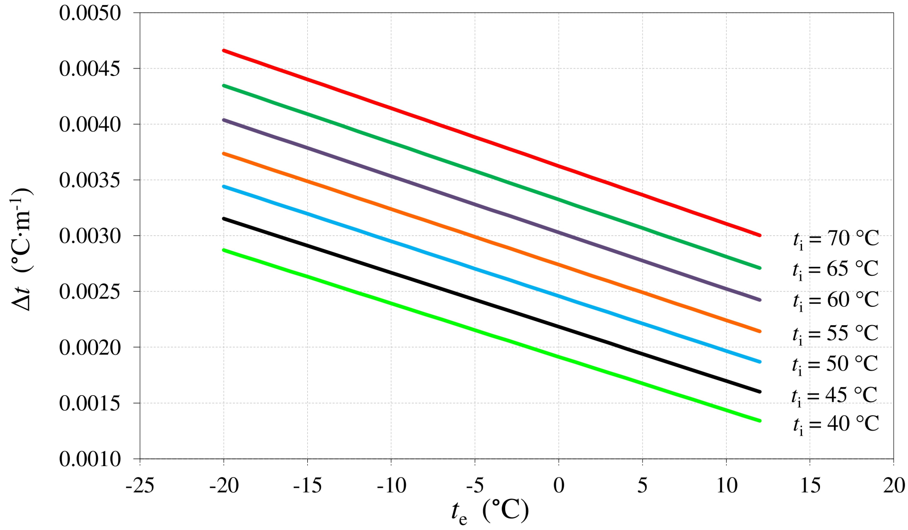

Figure 3. Dependence of Δt on the ambient temperature at different temperatures of transferred water and at $Q_V = 8.25\,\mathrm{m^3\,h^{-1}}$.

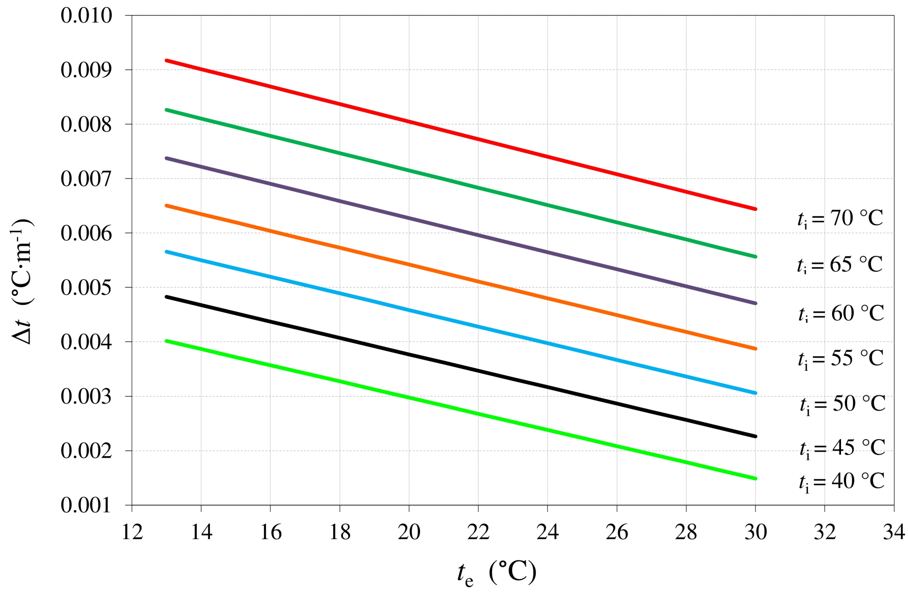

Figure 4. Dependence of Δt on the ambient temperature at different temperatures of transferred water and at $Q_V = 2.68\,\mathrm{m^3\,h^{-1}}$.

The nominal heat loss calculated by the balance method in the experimentally monitored network for both flow rates is demonstrated in Fig. 1. It is evident that the amount of water that has flowed does not influence the heat loss. The heat loss depends only on the temperature of the transferred water. The lower the temperature, the lower the heat loss will be.

On the basis of the calculated nominal heat loss in accordance with the analytic relations mentioned in the available literature, for piping above the ground, with the transferred water temperature between 70 °C and 40 °C, and a measured flow rate up to $23\,\mathrm{m^3\,h^{-1}}$, it was possible to set the temperature decrease Δt for a pipe one meter in length (Fig. 2).

In order to confront the heat loss results reached by the balance method and by the prepared model (23), the considered scope of the outside temperature was between minus 20 °C and plus 30 °C. This scope was defined on the basis of the measured outside temperatures in the course of one year.

Figure 3 demonstrates the course of Δt at a water flow rate of $Q_V = 8.25\,\mathrm{m^3\,h^{-1}}$ in dependence on the outside temperature during the heating period (−20 to +12 °C), with the transferred water temperature from 70 °C to 40 °C. Figure 4 demonstrates the course of Δt at a water flow rate of $Q_V = 2.68\,\mathrm{m^3\,h^{-1}}$ in dependence on the outside temperature outside the heating period (+13 °C to +30 °C).

5.2. Calculating the heat loss according to the model

To determine the heat loss according to model (23), it is necessary to know the precise values of the dimensionless arguments π_1, π_5 for the monitored piping,

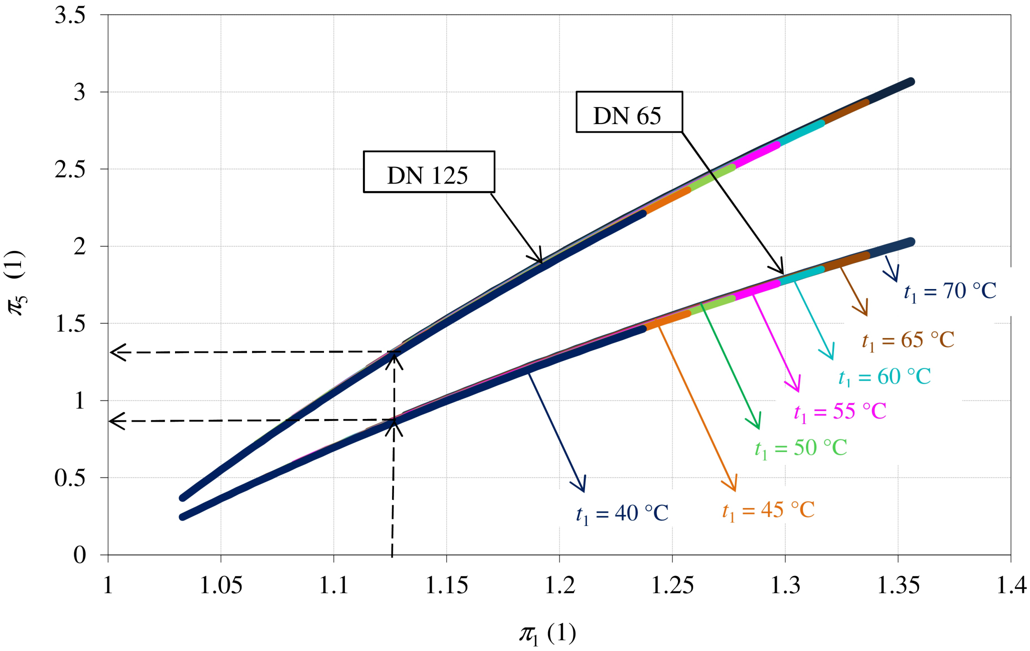

FIGURE 5. Dependency of the dimensionless arguments π_1 and π_5 for pipe DN125 and DN65.

the insulation thermal conductivity factor at the set temperature λ_{ins}, the actual temperature of the transferred water, and the ambient temperature.

The values of the dimensionless argument π_5 can be seen in dependence on π_1 in the diagram in Fig. 5. The diagram is prepared for pipes with nominal diameters DN125 and DN65. The precise value of the insulation thermal conductivity factor λ_{ins}, for relation (23) can be found in the tables at the set ambient temperature.

The procedure for calculating the heat loss is simple and fast, and it provides values comparable with those provided by the procedure applying complicated relations based on the theory of heat and mass transfer.

It is evident that the dimensionless argument π_5 reaches the same values of argument π_1 for pipeline DN65 (i.e., within the same range of temperatures of the transferred water, ambient temperatures, and the same thermal insulation), much lower values than for DN125. Thus the heat loss from a pipe one meter in length will be lower in pipe DN65 than in pipe DN125.

6. DISCUSSION

The two procedures described in this paper have been proved in a thermal network with a nominal diameter of DN 125.

According to the balance method, the heat loss was calculated by relation (8) with an achieved value of approx. 41.6 W m^{-1}. It was set at an ambient temperature of minus 15 °C. The incoming water temperature was 70 °C, and the water density at this temperature was 985 kg m^{-3}. The specific thermal capacity of water is 4186 J kg^{-1} K^{-1}, and the insulation conductivity factor is 0.041 W m^{-1} K^{-1}. The water flow rate is 8.25 m^3 h^{-1}. The decrease in temperature for piping one meter in length was 0.0044 K m^{-1} (after the measurement, as can be approximately found in the diagram in Fig. 3).

In determining the heat loss in accordance with model (24), the achieved value is approx. 41.8 W m^{-1}. This is based on the dimensionless argument π_1, the value of which is 1.329 for the same temperature of the incoming water, and the ambient temperature is calculated by Eq. (9). On the basis of the data in Fig. 5, the value of argument π_5 is 2.98. The transferred water temperature, the ambient and insulation temperature are in this case equal to the heat loss values determined by the balance method according to relation (8).

7. CONCLUSION

The heat loss model that has been elaborated is mainly suitable for heating network operators. It enables the heat loss to be determined simply and rapidly. The results provided by the model are comparable to the results obtained by the analytic procedure based on heat and mass transfer theory, or obtained by measurement.

For an experiment on any type of heating network that could represent a model, the dependency of dimensionless arguments π_1 to π_5 can be found for the changed conditions of the supplied water into the network and for a different ambient temperature. This method can easily be used for determining the specific heat loss by a graphic interpretation for all other networks of the same nominal diameter, with insulation of the same quality and thickness.

ACKNOWLEDGEMENTS

This paper was written with support from VEGA projects 1/0004/2014 MŠVV SR and SP2016/110-FMMI VŠB TUO.

References

[1] Durcansky, S. Papucik, Š., Jandacka, J., Holubcik, M., Nosek, R. Design of Heat Exchanger for Ericsson-Brayton Piston Engine. The Scientific World Journal. 2014, Article ID 138254 (2014):6 p., DOI:10.1155/2014/138254

[2] Raudensky, M. Heat transfer coefficient estimation by inverse conduction algorithm. International Journal of Numerical Methods for Heat & Fluid Flow. 3 (1993):257-266, DOI:10.1108/eb017530

[3] Brestovič, T., Jasminská, N., Čarnogurská, M., Puškár, M., Kelemen, M., Fiľo, M. Measuring of thermal characteristics for Peltier thermopile using calorimetric method. Measurement. 53 (2014):40-48, DOI:10.1016/j.measurement.2014.03.021

[4] Zavila, O., Herecová, L., Míček, D., Hejzlar, T. Numerical simulation of heavy and light pollutants motion as a tool of experimental data verification. Communication. 13, 2 (2011):37-43. ISSN: 1335- 4205.

[5] Danielewicz, J., Sniechowska, B., Sayegh, M. A., Fidorow, N., Jouhara, H. Three-dimensional numerical model of heat losses from district heating network pre-insulated pipes buried in the ground, Energy. 14 (2015):1-13, DOI:10.1016/j.energy.2015.07.012

[6] Nosek, R., Holubcik, M., Jandacka, J., Vician, P. Condenser optimization of heat pipe, Komunikacie, 16 (2014):62-66, Issue 3A, DOI:10.1016/j.enconman.2012.06.004

[7] Čarnogurská, M., Brestovič, T., Příhoda, M. Modelling of Nitrogen Oxides Formation Applying Dimensional Analysis, Chemical and Process Engineering. 32, 3 (2011):175-184. DOI:10.1007/s12206-010-0810-9

[8] Čarnogurská, M., Příhoda, M., Koško, M., Pyszko, R. Verification of pollutant creation model at dendromass combustion, Journal of Mechanical Science and Technology. 26, 12 (2012):4161-4169, DOI:10.1007/s12206-012-0877-6

[9] Zeleňáková, M., Čarnogurská, M. A dimensional analysis-based model for the prediction of nitrogen concentrations in Laborec River, Slovakia, Water and Environment Journal: Promoting Sustainable Solutions. 27, 2 (2013):284-291. DOI:10.1111/j.1747-6593.2012.00372.x

[10] Čarnogurská, M., Příhoda, M., Popčáková, D. Modelling the flow conditions in the tunnel and its reduced model. Journal of Mechanical Science and Technology. 24, 12 (2010):2479-2486, DOI:10.1007/s12206-010-0810-9

[11] Čarnogurská, M., Příhoda, M. Application of dimensional analysis at modelling of phenomena in power engineering. TU Košice, 2011, (In Slovak). ISBN 978-80-553-0699-5.

[12] Příhoda, M., Rédr, M. Fundamentals of thermal engineering. SNTL, 1991 Praha, ISBN: 80-03-00366-0, (In Czech).

[13] Lienhard IV, J. H., Lienhard V., J. H. A Heat Transfer Textbook. 4th ed. Phlogiston Press, Cambridge, 2012. ISBN: 978097138352.

[14] Jones, J. B. Hawkins, G. A. Engineering Thermodynamics. John Wiley, New York, 1986, ISBN: 13: 978-0471812029.

[15] Rohsenow, W., M., Hartnett, J. P., Cho, Y. I. Handbook of Heat Transfer. McGraw-Hill, 1998, ISBN: 0- 07-053555-8.

[16] Buckingham, E. On Physically Similar Systems; Illustrations of the Use of Dimensional Equations. Phys. Rev. 4 (1914):345-376. DOI:10.1103/PhysRev.4.345